2012 WISE Final Exam: Math Econ

December 27, 2012

Total points are 180. Total time is 180 minutes. Roughly one point per minute. Budget your time. Good Luck!

1. (10 points) For the following matrices A, find the general solution of $x_{n+1} = Ax_n$

$$A = \begin{pmatrix} 2 & 3 & 2 \\ -1 & 4 & 2 \\ 0 & 1 & 3 \end{pmatrix}$$

2. (25 points) $\min_{x,y}(x-3)^2 + (y-2)^2$ subject to $x^2 + y^2 \leq 5$, $x + 2y \leq 4$ and $x, y \geq 0$

3. (15 points) Is the sign of the polynomial $f(x_1, x_2) = x_1^2 - x_2^2 - 6x_1 + 2x_2 + 2x_1x_2 + 1$ determined when $x_1 + x_2 = 3$? Prove your result.

4. (15 points) Let f be a function defined on a convex subset U of R^n. Prove that f is concave if and only if its restriction to every line segment in U is a concave function of one variable.

5. (15 points) Prove that any Cobb-Douglas function $U(x, y) = x^a y^b$ is quasiconcave on R^2_+, where $a, b > 0$.

6. (20 points) Suppose that F is a C^1 function on an open convex subset U of R^n. Prove that F is quasiconcave on U if and only if $F(y) \geq F(x)$ implies that $DF(x)(y - x) \geq 0$

7. (15 points) Prove that any union of open sets is open; the finite intersection of open sets is open.

8. (15 points) Prove that $X_n \in R^m \to X$ if and only if $x_{in} \to x_i$ for all $i = 1, 2, \cdots, m$, where $X_n = (x_{1n}, x_{2n}, \cdots, x_{mn})'$ and $X = (x_1, x_2, \cdots, x_m)'$.

9. (15 points) Consider the system of two equations in three unknowns: $x + 2y + z = 5$, $3x^2yz = 12$. At the point $x = 2$, $y = 1$ and $z = 1$, why can we treat z as an exogenous variable and x and y are dependent variables? If z rises by an amount of Δz, express the changes of x and y in terms of Δz.

10. (20 points) Let f and h be C^2 functions: $R^2 \to R^1$. Consider the problem of maximizing f on the constraint set $C_h = \{(x, y) : h(x, y) = c\}$. Suppose that (x^*, y^*, μ^*) is the solution of Lagrangian conditions and satisfies

$$det \begin{pmatrix} 0 & \frac{\partial h}{\partial x} & \frac{\partial h}{\partial y} \\ \frac{\partial h}{\partial x} & \frac{\partial^2 L}{\partial x^2} & \frac{\partial^2 L}{\partial x \partial y} \\ \frac{\partial h}{\partial y} & \frac{\partial^2 L}{\partial x \partial y} & \frac{\partial^2 L}{\partial y^2} \end{pmatrix} > 0$$

Prove that (x^*, y^*) is a local maximizer of f.

11. (15 points) For an $n \times n$ matrix A, if $r_1, r_2, \cdots, r_n$ are distinct real eigenvalues and $v_1, v_2, \cdots, v_n$ are the corresponding eigenvectors, prove that $v_1, v_2, \cdots, v_n$ are linearly independent.