2011 Double Degree Final Exam: Math Econ

December 23, 2011

Total marks are 180. Total time is 180 minutes. Roughly one mark per minute. Budget your time. Good Luck!

1. (15 marks) Determine the definiteness of the symmetric matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 5 \\ 0 & 5 & 6 \end{pmatrix}$

2. (20 marks) Determine the definiteness of $Q(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 + 4x_1x_3 - 2x_1x_2$ subject to $x_1 + x_2 + x_3 = 0$

3. (15 marks) Find the max and min of $f(x, y, z) = x + y + z^2$ subject to $x^2 + y^2 + z^2 = 1$ and $y = 0$.

4. Maximize $f(x, y) = 3xy - x^3$ subject to $2x - y = -5$, $5x + 2y \geq 37$, $x \geq 0$ and $y \geq 0$.

   (a) (25 marks) Solve the above question and derive the maximum objection function value $f(x^*, y^*)$.

   (b) (15 marks) How would $f(x^*, y^*)$ change if the first equality constraint, $2x - y = -5$, changes to $2x - y = -4.5$

5. (15 marks) Let $f(x; a)$ be a C^1 function of $x \in R^n$ and the scalar a. For each choice of the parameter a, consider the unconstrained maximization problem: max $f(x; a)$ with respect to x. Let $x^*(a)$ be a solution of this problem. Suppose that $x^*(a)$ is a C^1 function of a. Prove that:

$$\frac{d}{da} f(x^*(a), a) = \frac{\partial}{\partial a} f(x^*(a), a)$$

6. (15 marks) Let $f(x)$ be a C^1 homogeneous function of degree k on R^n_+. Prove $x^T \triangledown f(x) = kf(x)$

7. Consider the function $f(x, y) = x^4 + x^2y^2 + y^4 - 3x - 8y$

   (a) (15 marks) Is $f(x, y)$ homogeneous or homothetic ? Prove your claim.

   (b) (15 marks) Is $f(x, y)$ concave or convex? Prove your claim.

8. (15 marks) Write down the definition of quasiconcave functions. And illustrate one example that is a quasiconcave function but not a concave function.

9. (15 marks) Prove that any monotone transformation of a concave function is quasiconcave.