Mathematical Economics: Lecture 3

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October 7, 2012

Outline

Example of Graphing

Example 3.1

Consider the cubic function $f(x) = x^3 - 3x$. Try to graph this function

Example of Graphing

- $f'(x) = 3x^2 - 3 = 3(x-1)(x+1)$
- $f'(x) = 0$ at $x = -1, +1$. **critical points** of f.
- the corresponding points: (-1,2) and (1,-2).

Example of Graphing

- check the sign of f' in the following intervals $J_1 = (-\infty, -1)$, $J_2 = (-1. + 1)$, *and* $J_3 = (+1, +\infty)$
- (a) f is increasing on J_1;
  (b) f is decreasing on J_2;
  (c) f is increasing on J_3.
- Figure 3.3.

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General Rule

- find critical points: $f'(x) = 0$ or f' is not defined
- evaluate the critical points
- check the sign of f' on the intervals between two critical points
- graph $f' > 0$, $f' < 0$

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Second Derivatives and Convexity

- convex: $f'' > 0$
- concave: $f'' < 0$
- inflection points: $f'' = 0$
- make graphing more precisely

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Tails and Horizontal Asymptotes

- graph $\frac{9x^7+8x^5+5\sqrt{x}+3}{10x^2+4x+5}$ for $x \in (300, 400)$

Tails and Horizontal Asymptotes

- leading term : the monomial of highest degree
- For $|x|$ very large, the graph of a polynomial $f(x) = a_0x^k + a_1x^{k-1} + \cdots + a_{k_1}x + a_k$ is determined completed by its leading term a_0x^k

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Tails and Horizontal Asymptotes

- A general rational function $g(x) = \frac{a_0x^k + a_1x^{k-1} + \cdots + a_{k_1}x + a_k}{b_0x^m + b_1x^{m-1} + \cdots + b_{m_1}x + b_m}$ mirrors the behavior of the monomial $l(x) = \frac{a_0x^k}{b_0x^m}$

Maxima and Minima

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- for example: $f(x) = x$ $x \in [0, 1]$. 1 and 0 are boundary max and boundary min.

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Maxima and Minima: Theorems

- **Theorem 3.3**: If x_0 is an interior max or min of f, then x_0 is a critical point of f. This means $f'(x_0) = 0$. Example: $f(x) = x^2$.

Maxima and Minima: Theorems

(Local) **Theorem 3.4**:

- (a) $f'(x_0) = 0$ $f''(x_0) < 0$, x_0 max
- (b) $f'(x_0) = 0$ $f''(x_0) > 0$, x_0 min
- (c) $f'(x_0) = 0$ $f''(x_0) = 0$, x_0 no conclusion

Maxima and Minima: Theorems

- (Global) **Theorem 3.5**: x_0 is a local max and is the only critical point, then x_0 is the global max.

Example

Example 3.5 Find the local max and mins of

$$f(x) = x^4 - 4x^3 + 4x^2 + 4$$

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$f''(x) = 12x^2 - 24x + 8$, at these three points:

$$f''(0) = 8 > 0$$
$$f''(1) = -4 < 0$$
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by Theorem 3.4, $x = 0$ and $x = 2$ are local mins of f and $x = 1$ is a local max

Figure 3.15

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