Mathematical Economics: Lecture 5

Yu Ren

WISE, Xiamen University

October 10, 2012

Outline

New Section

Chapter 10: Euclidean Spaces

Points in Euclidean Space

- R^1 consists of single numbers, $\{x_i\}$
- R^2 consists of order pairs $\{(x_i, x_j)\}$
- R^n, $\{(x_i, \cdots, x_{i+n-1})\}$

Points in Euclidean Space

- A space with notions of *distance* and *angle* that obey the Euclidean relationships is called an Euclidean space

Vectors in Euclidean Space

- In R^2, **displacement** (x,y) means: move x units to the right and y units up from your current location. Figure 10.6 (P 203)

Vectors in Euclidean Space

- In R^n, the displacement from the point $p(a_1, a_2, \cdots, a_n)$ to the point $q(b_1, b_2, \cdots, b_n)$ is

$$\overrightarrow{pq} = (b_1 - a_1, b_2 - a_2, \cdots, b_n - a_n)$$

The Algebra of Vectors

- Addition and Subtraction:
$$\underbrace{(x_1, x_2) + (y_1, y_2) = (y_1, y_2) + (x_1, x_2)}_{commutative} =$$
$$(x_1 + y_1, x_2 + y_2)$$
- Geometric interpretation: Figure 10.8 10.9 (Page 205, Page 206)
- Scalar multiplication:
$$r \cdot X = (rx_1, rx_2, \cdots, rx_n),$$
$$(r + s)X = rX + sX \quad r(U + V) = rU + rV$$

Length

- Notation: line segment $\bar{PQ}$; vector $\overrightarrow{PQ}$; length $||\bar{PQ}||$
- $P = (a_1, b_1)$, $Q = (a_2, b_2)$,

Length

- $||\bar{PQ}|| = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$
- $||Q - P|| = ||\overrightarrow{PQ}|| = ?$

Inner Product

- Inner product: let $U = (u_1, \cdots, u_n)$ and $V = (v_1, \cdots, v_n)$. The Euclidean inner product of U and V, written $U \cdot V$, is the number:

$$U \cdot V = u_1 v_1 + u_2 v_2 + \cdots + u_n v_n$$

Properties of Inner Product

- $U \cdot V = V \cdot U$
- $U \cdot (V + W) = U \cdot V + U \cdot W$
- $U \cdot (rV) = r(U \cdot V) = (rU) \cdot V$
- $U \cdot U \geq 0$
- $(U + V) \cdot (U + V) = U \cdot U + 2(U \cdot V) + V \cdot V$

Properties of Inner Product

- $||U|| = \sqrt{U \cdot U}$
- $||U - V|| = \sqrt{(U - V) \cdot (U - V)}$
- $||U + V|| \leq ||U|| + ||V||$
- $|\ ||U|| - ||V||\ | \leq ||U - V||$

Angle

- $U \cdot V = ||U||||V||\cos\theta$

Angle

The angle θ between U and V is

- (a) acute if $U \cdot V > 0$
- (b) obtuse if $U \cdot V < 0$
- (c) right $U \cdot V = 0$, call U and V are orthogonal.

Lines and Planes

- line: $X(t) = X_0 + tV$ Figure 10.26. (page 223)
  Example: $X_0 = (1, 2)$ $V = (1, 1)$
- plane:$X = P + sV + tW$ Figure 10.27 10.28 (page 226)
- hyperplane: a hyperplane in R^n is $a_1x_1 + a_2x_2 + \cdots + a_nx_n = d$

Economic Application

- Commodity space: $\{(x_1, \cdots, x_n) : x_1 \geq 0, \cdots, x_n \geq 0)\}$, commodity bundle
- Budget set: $P \cdot X \leq I$ Figure 10.31 (Page 232)
- Probability simplex : $P_n = \{(p_1, \cdots, p_n) : p_i \geq 0 \text{ and } p_1 + \cdots + p_n = 1\}$

Economic Application

- Commodity space: $\{(x_1, \cdots, x_n) : x_1 \geq 0, \cdots, x_n \geq 0)\}$, commodity bundle
- Budget set: $P \cdot X \leq I$ Figure 10.31 (Page 232)
- Probability simplex : $P_n = \{(p_1, \cdots, p_n) : p_i \geq 0 \text{ and } p_1 + \cdots + p_n = 1\}$

Economic Application

- Commodity space: $\{(x_1, \cdots, x_n) : x_1 \geq 0, \cdots, x_n \geq 0)\}$, commodity bundle
- Budget set: $P \cdot X \leq I$ Figure 10.31 (Page 232)
- Probability simplex : $P_n = \{(p_1, \cdots, p_n) : p_i \geq 0 \text{ and } p_1 + \cdots + p_n = 1\}$