

2012 WISE Midterm Exam: Mathematical Economics

November 11, 2012

There are 10 questions in this exam. The questions are equally weighted.

Good Luck!

1. Calculate the inverse matrix of

$$\begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix}$$

2. For two vectors u and v in R^n , prove that $\|u + v\| \leq \|u\| + \|v\|$
3. Consider the following three vectors: $v_1 = (\lambda, -0.5, -0.5)$, $v_2 = (-0.5, \lambda, -0.5)$, $v_3 = (-0.5, -0.5, \lambda)$. If possible, find a value for λ (a scalar) such that the three vectors are linearly dependent. If no value is possible, explain why not.
4. Prove that if the columns of B are linearly dependent then so are the columns of AB .
5. In R^3 space, let $v_1 = (1, 3, 2)$, $v_2 = (7, 4, 5)$, $v_3 = (11, -1, 4)$. And $W = L[v_1, v_2, v_3]$ denotes the set spanned by v_1, v_2, v_3 . Please find a basis for the set W , which contains NO v_1, v_2 or v_3 .
6. Prove that $\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$
7. Prove that $\{x_m\} \subseteq R^n$ converges to x if and only if every subsequence of $\{x_m\}$ converges to x .
8. Prove that S is closed if and only if S^c is open.
9. Prove that $(\bigcup_{i \in I} A_i)^c = \bigcap_{i \in I} A_i^c$

10. Consider the sets $A = \bigcup_{n=1}^{\infty} [1 + \frac{1}{n}, 10 - \frac{1}{n}]$. Show the interior, the closure, the boundary of A and explain your results. In addition, indicate whether A is open or closed sets. Why?